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Bálint Tóth and Benedek Valkó

Superdiffusive bounds on self-repellent precesses in $d = 2$ — extended abstract

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Centre international de rencontres mathématiques
U.M.S. 822 C.N.R.S./S.M.F.
Luminy (Marseille) FRANCE
Superdiffusive bounds on self-repellent precesses in $d=2$ — extended abstract

Bálint Tóth and Benedek Valkó

Abstract

We prove superdiffusivity with multiplicative logarithmic corrections for a class of models of random walks and diffusions with long memory. The family of models includes the “true” (or “myopic”) self-avoiding random walk, self-repelling Durrett-Rogers polymer model and diffusion in the curl-field of (mollified) massless free Gaussian field in 2D. We adapt methods developed in the context of bulk diffusion of ASEP by Landim-Quastel-Salmhofer-Yau (2004).

We study the long time asymptotics of the self repelling Brownian polymer process (SRBP) in $\mathbb{R}^d$ defined by the SDE

$$dX_t = dB_t(\eta_0 - \text{grad } V \ast l_t)(X(t))dt,$$

where $B_t$ is standard Brownian motion in $\mathbb{R}^d$, $\eta_0 : \mathbb{R}^d \to \mathbb{R}^d$ is a gradient vector field with sufficient regularity,

$$l_t(A) := \{ s \in [0,t] : X_s \in A \}$$

is the occupation time measure of the process $X_t$, and $V : \mathbb{R}^d \to [0,\infty)$ is a $C^\infty$, spherically symmetric approximate identity with sufficiently fast decay at infinity. It is assumed that $V(\cdot)$ is positive definite:

$$\hat{V}(p) = \int_{\mathbb{R}^2} e^{ip \cdot x} V(x) dx \geq 0.$$

The process is pushed by the negative gradient of its own local time, mollified by convoluting with $V$. The process was introduced by Durrett and Rogers in [2] and further investigated in a series of probability papers. For a survey and complete list of references see [6]. It is phenomenologically similar to the so-called true self-avoiding random walk (TSAW) which arose in the physics literature initiated by Amit, Parisi and Peliti in [1] and further investigated in a series of physics papers. For a survey and complete list of references see [10].

Conjectures based on scaling and renormalization group arguments regarding this family of models are the following (see e.g. [1]):

- In $d=1$: $X(t) \sim t^{2/3}$ with intricate, non-Gaussian scaling limit.
- In $d=2$: $X(t) \sim t^{1/2}(\log t)^{1/4}$ and Gaussian (that is Wiener) scaling limit expected.
- In $d \geq 3$: $X(t) \sim t^{1/2}$ with Gaussian (i.e. Wiener) scaling limit expected.

Some of these conjectures had been proven or at least partially settled. For results in $d=1$ see [9], [13], [12], [8] and the survey [10]. For results in $d \geq 3$ see [3]. The present lecture concentrates on recent results on the $d=2$ case. The complete results and proofs will appear in [11].

The natural framework of formulation of the problem and results is the environment seen by the random walker. Let

$$\eta_t(x) := (\eta_0 - \text{grad } V \ast l_t)(X(t)).$$

Then $t \mapsto \eta_t$ is a Markov process with continuous sample paths in the function space

$$\Omega = \{ \omega \in C^\infty(\mathbb{R}^2 \to \mathbb{R}^2) : \text{rot } \omega \equiv 0, \| \omega \|_{k,m,r} < \infty \}.$$
where $\| \omega \|_{k,m,r}$ are the seminorms
\[ \| \omega \|_{k,m,r} = \sup_{x \in \mathbb{R}^d} (1 + |x|)^{-1/r} \left| \partial_{m_1}^{m_1} \cdots \partial_{m_d}^{m_d} \omega_k(x) \right| \]
defined for $k = 1, 2$, multiindices $m = (m_1, \ldots, m_d)$, $m_j \geq 0$, and $r \geq 1$.

It was proved in [8] (for $d = 1$) and [3] (for arbitrary $d$) that the Gaussian measure $\pi$ on $\Omega$ defined by the covariances
\[ \int_{\Omega} \omega_k(x) d\pi(\omega) = 0, \quad K_{kl}(x-y), \quad \text{with} \quad \hat{K}_{kl}(p) = \frac{\partial_k \partial_l}{|p|^2} \hat{\varphi}(p) \]
is stationary and ergodic for the Markov process $\eta_t$.

The random vector field $\omega$ distributed according to $\pi$ the gradient of the massless free Gaussian field smeared out by convolution with $U$, where $U * U = V$.

Let
\[ \hat{E}(\lambda) := \int_0^\infty e^{-\lambda t} E \left( |X_t|^2 \right) dt. \]

The main result of [11], reported in this lecture is the following theorem.

**Theorem 1.** Consider the process defined by the stochastic differential equation (0.1) in $\mathbb{R}^2$ and let the initial vector field $\eta_0$ be sampled from the stationary distribution $\pi$. Then there exist constants $0 < C_1, C_2 < \infty$ such that for $0 < \lambda < 1$ the following bounds hold
\[ C_1 \log |\log \lambda| \leq \lambda^2 \hat{E}(\lambda) \leq C_2 |\log \lambda| \]

**Remarks:**
1. Modulo Tauberian inversion, these bounds mean in real time
\[ C_3 \lambda \log \lambda \leq E \left( |X(t)|^2 \right) \leq C_4 \lambda \log \lambda, \]
with $0 < C_3, C_4 < \infty$ and for $t$ sufficiently large.
2. The upper bound is straightforward, it follows directly form estimates on diffusion in random scenery. The superdiffusive lower bound is the main result.

The proof of Theorem 1 follows the main lines of [5]. See also [4]. However on the computational level there are notable differences. For similar recent results referring to second class particle motion in various asymmetric exclusion processes see also [7].

Full proofs are available in [11].

**References**


